Translation:Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies/Section VI

1 Experiments whose results cannot be explained without further ado.

1.1 The rotation of the polarization plane.

§ 84. As the equations of motion of light for an isotropic body that has *not* the same properties as its mirror image, we have to assume by the considerations of the 4th section:

where by $\mathfrak{E}, \mathfrak{d}, \mathfrak{H}$ and $\mathfrak{H}'$ we have to understand averages.

We now want to presuppose, that the velocity $\mathfrak{p}$ would have the direction of the *x*-axis, and to study the propagation of plane waves, whose normal coincides with that axis as well.

§ 85. To find a particular solution of the equations corresponding to such waves, we put

$$\mathfrak{H}_x = 0,\ \mathfrak{H}_y = ae^{nt-mx},\ \mathfrak{H}_z = \nu\, \mathfrak{H}_y$$

where *a, ν, n* and *m* are constants. Already by that, the condition (II_e) is satisfied.

Now, the equation (IV_e) will be satisfied by us, by putting

$$\mathfrak{E}_x = 0,\ \mathfrak{E}_y = \tfrac{n}{m}\mathfrak{H}_z,\ \mathfrak{E}_z = -\tfrac{n}{m}\mathfrak{H}_y\ ,$$

and then it follows from (V_e), (VI_e) and (III_e), one after the other,

whose latter values are also in agreement with condition (I_c).

Eventually we derive from (X)

and then we only have to satisfy condition (XI).

The first one of the herein summarized relation gives nothing new, while the second and third ones read:

and

Now, since by the reported formulas

$$\mathfrak{E}_y = -\nu\mathfrak{E}_z \text{ und } \mathfrak{M}_y = -\nu\mathfrak{M}_z$$

it can be written for (110) and (111)

$$\nu(\mathfrak{E}_z - \sigma\mathfrak{M}_z) = j\ \tfrac{\partial\mathfrak{M}_z}{\partial x} - k\dot{\mathfrak{M}}_z\mathfrak{p}_x\ ,$$

and

$$\mathfrak{E}_z - \sigma\mathfrak{M}_z = -\nu\left(j\ \tfrac{\partial\mathfrak{M}_z}{\partial x} - k\dot{\mathfrak{M}}_z\mathfrak{p}_x\right)\ ,$$

thus at first we find

$$\nu^2 = -1,\ \nu = \pm i\ ,$$

and furthermore

Now, if σ, j, k and *n* are given, we can determine *m* from this equation, namely we obtain *two* values, depending on whether we apply the above, or the below sign.

§ 86. We put

$$n = in',\ m = im'\ ,$$

by that, equation (112) is transformed into

from which two *real* values are given from m' , which we want to denote by m_1' and m_2' .

For $\nu = +i,\ m' = m_1'$, it becomes now

$$\mathfrak{H}_y = ae^{i(n't-m_1'x)},\ \mathfrak{H}_z = iae^{i(n't-m_1'x)}\ ,$$

and for $\nu = -i,\ m' = m_2'$,

$$\mathfrak{H}_y = ae^{i(n't-m_2'x)},\ \mathfrak{H}_z = -iae^{i(n't-m_2'x)}\ .$$

If we eventually take the *real* parts, we arrive at the following two particular solutions

which obviously represent two opposite, circular-polarized light beams of propagation velocities n'/m_1' and n'/m_2' .

The composition of these states of motion leads in a known way to a beam of linear-polarized light, whose oscillation direction gets rotated. Namely, addition of the values (114) and (115) gives the solution

The rotation ω of the polarisation plane related to unit volume, consequently amounts

$$\omega = \tfrac{1}{2}(m_1' - m_2') \, .$$

§ 87. If we replace in equation (113), $\mp j$ by α , and $\mp k\mathfrak{p}_x$ by β , it follows

$$4\pi V^2 \tfrac{n'}{m'} = (\sigma + \alpha m' + \beta n') \left(V^2 \tfrac{m'}{n'} - \tfrac{n'}{m'} - 2\mathfrak{p}_x \right) .$$

Since the terms with α, β and $\mathfrak{p}_x$ are in any case very small, the value of *m* following from it, can be represented by a row that progresses with respect to the powers of α, β and $\mathfrak{p}_x$. The first term independent of these magnitudes, has the value

$$m_0' = n' \sqrt{\tfrac{4\pi}{\sigma} + \tfrac{1}{V^2}} \, ,$$

and then we also find

where we didn't calculated the three latter terms more closely, and we have neglected all higher powers of α and β , as well as all terms that include $\mathfrak{p}_x^2$. To these latter ones, also the terms with β^2 and $\beta\mathfrak{p}_x$ do belong, since $\beta = \mp k\mathfrak{p}_x$.

Now, we obtain m_1' , or m_2' , depending on whether we put $\alpha = -j, \ \beta = -k\mathfrak{p}_x$, or $\alpha = +j, \ \beta = +k\mathfrak{p}_x$. The sought rotation of the polarization consequently becomes

$$\omega = \tfrac{2\pi}{\sigma^2} n'^2 \left(1 + \tfrac{n'}{m_0'} \tfrac{\mathfrak{p}_x}{V^2} \right) j + \tfrac{2\pi}{\sigma^2} \tfrac{n'^3}{m_0'} \mathfrak{p}_x k \, ,$$

or, when we denote the propagation velocity $\frac{n'}{m_0'}$ by *W*,

$$\omega = \tfrac{2\pi}{\sigma^2} n'^2 \left(1 + \tfrac{W\mathfrak{p}_x}{V^2} \right) j + \tfrac{2\pi}{\sigma^2} n'^2 W \mathfrak{p}_x k \, .$$

The natural rotation of the polarization plane in stationary bodies would consequently be

if we were allowed to consider as constant σ and *j*, then it would be, as it follows from the meaning of n' , proportional to the square of the oscillation time. It's known that all bodies deviate more or less from this law; but we already know, that σ is changing with the duration of oscillation, and *j* probably might depend on it as well.

The translation has two influences by our equation. First, it changes the already existing rotation in the ratio

and furthermore it additionally causes a rotation.

The theory cannot give a relation between this value and (116); probably such a relation doesn't exist at all, and cases could exist, in which *j* is very small, while *k* nevertheless has a noticeable value.

By the way, it is probably not required to be remarked, that the phenomena represented by (118) are similar to the polarization in so far, as it also only arises by an *exterior* influence, namely by the translation, and most strongly emerges, when this influence has the direction of the light rays.

§ 88. Experiments on the rotation of the polarization plane at different orientation, as far as I know, were only undertaken by Mascart[1] He was unable to conclude a change of rotation with respect to quartz, when the light rays have, on one hand, the direction of Earth's motion, and on the other hand, the opposite direction. For the observation it had to be concluded, that the change in any case didn't amount the 20000th part of the rotation, and as regards a certain direction of the light rays, the rotation was altered by Earth's motion by less than 1/40000.

Due to the lack of a theory applicable for anisotropic bodies, we maybe also apply the above reported formulas to quartz. Now, since the refractive index is 1,55, and $\mathfrak{p}_x/V = 1/10000$, then the value of the second member in (117) becomes 0,000064. The change of rotation caused by that, could not have been overlooked by Mascart, and thus his negative result can only by explained by the assumption, that, in the formula for ω , *k* has a value comparable with j/V^2 , and the opposite sign of *j*.

Now, whether (for quartz and other bodies) the two terms containing $\mathfrak{p}_x$ will mutually be cancelled, or whether an observable influence of Earth's motion remains finally, has to be decided by additional investigations.

1.2 The interference experiment of Michelson.

§ 89. As it was first noticed by Maxwell, and which follows from a very simple calculation, the time required by a light ray to travel forth and back between two points *A* and *B* must change, as soon as these points are subject to a common displacement, without dragging the aether. Although the variation is a magnitude of second order, it is nevertheless big enough that it can be demonstrated by means of a sensitive interference method.

The experiment was executed by Michelson in the year 1881.[2] His apparatus, a kind of interference-refractor, had two equally long, horizontal, mutually perpendicular arms *P* and *Q*, and from the two mutually interfering light beams, one went forth and back along arm *P* and the other one along arm *Q*. The whole instrument, including the light source and the observation device, could be rotated around a vertical axis, and especially the two locations come into consideration, at which arm *P* or arm *Q* had (so far as possible) the direction of Earth's motion. Now, during the rotation from one "main-position" into the other, a displacement of the interference fringes was expected on the basis of Fresnel's theory.

However, the change in this displacement caused by the variation of the propagation times — we want to call it Maxwell's displacement for sake of brevity — was found, and thus Michelson thought that he is allowed to conclude that the aether wouldn't remain at rest when the Earth is moving, a conclusion however, whose correctness was soon questioned. By inadvertence, Michelson has estimated the change of the phase differences as expected by

the theory, to double of the correct value; if we correct this error, we arrive at displacements, which just could be hidden by the observational errors.

Together with Morley, Michelson has started again the investigation,[3] where (to increase the sensitivity) he let reflect every light beam by some mirrors back and forth. This artifice gave the same advantage, as if the arms of the earlier apparatus would have been considerably extended. The mirror was carried by a heavy stone plate, that floated on mercury and thus was easily rotatable. Altogether, every beam had to traverse a path of 22 meters, and by Fresnel's theory, when passing from one main-position to the other, a displacement of 0,4 of the fringe-distance was to be expected. Nevertheless, during the rotation only displacements of at most 0,02 of the fringe-distance were obtained; they probably might stem from observational errors.

Now, is it allowed to assume on the basis of this result, that the aether shares the motion of Earth, and thus Stokes' aberration theory is the correct one? The difficulties, with which this theory is confronted when explaining aberration, seem too great to me as for having that opinion, so I rather should try to remove the contradiction between Fresnel's theory and Michelson's result. Indeed this can be achieved by means of a hypothesis, which I already have spoken out some time ago,[4] and to which, as I found out later, also Fitzgerald arrived.[5] Of which the hypothesis consists, shall be shown in the next §.

§ 90. For simplification we want to assume, that we would work with an instrument as that during the first experiments, and that with respect to one main-position, the arm *P* coincides exactly with the direction of Earth's motion. Let $\mathfrak{p}$ be the velocity of this motion, and *L* the length of every arm, thus *2L* the path of the light rays. Then by the theory [6], the translation causes that the time, in which one light-beam travels forth and back along *P*, is longer by

$$L \cdot \frac{\mathfrak{p}^2}{V^3}$$

than the time, in which the other beam completes its path. Exactly this difference would also exist, when (without that the translation would have an influence) arm *P* would be longer by

$$L \cdot \frac{\mathfrak{p}^2}{2V^2}$$

than arm *Q*. Similar things are true for the second main-position.

Thus we see, that the phase difference expected by the theory could also arise, when (during the rotation of the apparatus) sometimes one, sometimes the other arm would have the greater length. From that if follows, that they can be compensated by opposite variations of the dimensions.

If we assume, that the arm lying in the direction of Earth's motion, is shorter by

$$L \cdot \frac{\mathfrak{p}^2}{2V^2}$$

than the other one, and simultaneously the translation would have an influence which follows from Fresnel's theory, then the result of Michelson's experiment is fully explained.

Consequently we have to imagine, that the motion of a rigid body, *e.g.* a brass rod or of the stone plate used in later experiments, would have an influence on the dimensions throughout the aether, which, depending on the orientation of the body with respect to the direction of motion, is different. *E.g*, if the dimensions parallel to the direction of motion would be changed in the ratio of 1 to $1 + \delta$, and the dimensions perpendicular to them by a ratio of 1 to $1 + \epsilon$, than it should be

Here, the value of one of the magnitudes δ and ϵ would remain undetermined. It could be $\epsilon = 0,\ \delta = -\frac{\mathfrak{p}^2}{2V^2}$, but also $\epsilon = \frac{\mathfrak{p}^2}{2V^2},\ \delta = 0$, or $\epsilon = \frac{\mathfrak{p}^2}{4V^2}$, and $\delta = -\frac{\mathfrak{p}^2}{4V^2}$.

§ 91. As strange as this hypothesis would appear at first sight, nevertheless one must admit that it's not so far off, as soon as we assume that also the molecular forces, similarly as we now definitely can say it of the electrical and magnetic forces, are transmitted through the aether. If this is so, then the translation will change the action between two molecules or atoms most likely in a similar way, as the attraction or repulsion between charged particles. Now, since the shape and the dimensions of a fixed body are, in the last instance, determined by the intensity of the molecular effects, then also a change of the dimensions is inevitable.

Thus from a theoretical perspective there is no objection to the hypothesis. As regards the experimental confirmation, it is to be noticed at first, that the relevant elongations and contractions are extremely small. We have $\mathfrak{p}^2/V^2 = 10^{-8}$, and thus (in case we put $\epsilon = 0$) the contraction of one diameter of Earth would amount ca. 6,5 c.M. The length of a meter rod, however, changes by $^1/_{200}$ Micron (when we bring it from one main-position into the other). If we would like to observe magnitudes so small, then we probably can hope to succeed only by an interference method. Thus we would have to work with two mutually perpendicular rods, and of two mutually interfering light beams, let one travel back and forth with respect to the first rod, and the other with respect to the second rod. By that we come again, however, to Michelson's experiment, and we wouldn't observe any displacement of the fringes during the rotation. In reverse as we have expressed it earlier, we could say now, that the displacement stemming from the changes of length, is compensated by Maxwell's displacement.

§ 92. It is noteworthy, the we are led exactly to the above presupposed changes of dimensions, when we *first* (without consideration of the molecular motion) assume, that

in a rigid body which remains at its own, the forces, attractions or repulsions which act on an arbitrary molecule, are mutually in equilibrium, and *second* — for which, however, there is no reason — when we apply to these molecular forces the law which we have derived in § 23 for the electrostatic actions. If we understand by S_1 and S_2 , not two systems of charged particles as in that paragraph, but two systems of molecules, — the second at rest and the first with the velocity $\mathfrak{p}$ in the direction of the x -axis —, between whose dimensions the relation given early exists, and if we assume, that in both systems the x-components of the forces are the same, but the y- and z-components are mutually different by the factors given in § 23, then it is clear, that the forces in S_1 will be mutually canceled, as soon as this happens in S_2 . Consequently, if S_2 is the state of equilibrium of a stationary, rigid body, then in S_1 the molecules have exactly those positions, in which they can remain under the influence of translation. The displacement would of course cause this configuration by itself, and thus by (24) it would cause a contraction in the direction of motion in the ratio of 1 to $\sqrt{1-\frac{\mathfrak{p}^2}{V^2}}$. This leads to the values

$$\delta=-\frac{\mathfrak{p}^2}{2V^2},\ \epsilon=0\ ,$$

which is in agreement with (119).

In reality the molecules of a body are not at rest, but there exists a stationary motion in every "equilibrium state". As to how this condition is of influence as regards the considered phenomenon, may remain undecided; in any case, due to inevitable observational errors, the experiments of Michelson and Morley let remain a considerable wide margin for the values of δ and ϵ .

1.3 The polarization experiments of Fizeau.

§ 93. In the oblique passage of a polarized light beam through a glass plate, the azimuth of the polarization changes in general, namely this phenomenon is depending on the nature of the plate, so that the increase or decrease of its refractive index is followed by a rotation of the polarization plane of the emanating light. This fact was the starting point for the experiments with glass columns, executed by Fizeau[7], whose results deserve our attention to a high degree. The apparatus employed, consisted of a polarized prism, a number of glass columns located after one another, and an analyzer. At the time of solstice, mostly at noon, the devise was turned at first with the polarizator into the east, and with the analyzer into the west, then they were brought into the opposite direction, while in the whole time, a beam of light rays was sent through by means of appropriately located mirrors. Although some irregularities showed up in the settings of the analyzer, yet altogether, a constant difference between the obtained readings for both locations seemed to exist. When I developed the present theory, I hoped at first to be able to explain this difference, but soon I found myself disappointed in my expectation. If the equations developed by me are correct, then an influence, as the one expected by Fizeau, cannot exist. The prove for that should be given by the next paragraph.

§ 94. Since we were working with white light, and the rotation of the polarization plane in the glass columns is not the same for all colors, so it was necessary to compensate the dispersion that arose from it. For that, circular-polarizing fluids were used, *e.g.* lemon oil or turpentine, and sometimes thin quartz plates that were cut perpendicular to the axis. For simplicity, we want to assume however, that light is homogeneous and therefore that no such substances are available in the apparatus. The theorem derived in § 59, is then readily applicable as it applies to an arbitrary system of refractive or birefringent bodies.

Now, an ideal experiment with respect to a stationary earth shall be compared with a real experiment, in which the apparatus in relation to Earth's motion is oriented in an arbitrary way. In the first case, the polarizer shall receive rays from the direction *s* and the oscillation period *T*; here, we imagine the analyzer thus placed that it does not transmit light. In the latter case the "corresponding" state of motion (§ 59) shall exist. For that, the incident light must have the relative oscillation period T(§ 60 a), and still have the ray-direction *s* (§ 60, b). Behind the analyzer, it will be dark again (§ 60, *b*), and we may therefore conclude:

Which direction Earth's motion may have, whether from the polarizer to the analyzer, or vice versa, light will always be erased at the presupposed position of the analyzer, as long as nothing is changed as regards the relative period of oscillation and the direction of the rays in relation to the apparatus.

Obviously, these conditions would have been met by the experiments, when the sun would have emitted white light. The relative oscillation period would thus have been as it is required by Doppler's law, and namely at each position of the apparatus. As for the direction of the beams in relation to the glass columns, it has probably not been exactly the same with respect to the various readings; however, this has not caused an error, since an influence of a small directional change of the incident light would hardly have been overlooked by the observer.

§ 95. The phenomenon that was expected by Fizeau and what he really believed to have observed, would have to occur even when using homogeneous light. Thus, here we come to a contradiction, that I can not solve. A source of error, of which one could say that it would have caused the differences in the analyzer locations, I could not discover. The activated circular-polarizing substances were probably a little too thick to allow a prominent influence of Earth's motion considered in § 87. Nor is it possible to think of an effect of terrestrial magnetism. The only thing might yet be, that the two mirrors located east and

west of the apparatus, have not always received light of the same nature. Namely, to reflect rays of the sun, sometimes by one, and sometimes by the other mirror, the heliostat had to have different positions; between the angles at which he threw back light in both cases, there was a difference depending on the position of the sun, and we know that light reflected by a metal surface, has not the same composition in all directions of incidence. . Since the mutual position of the mirror was not known to me, I was unable to calculate the influence of this error, and it was only possible to estimate it only superficially, by making an appropriate assumption on that location, and by applying the usual formulas for the metal reflection. In this way, the calculation, however, led to a difference in the analyzer positions with respect to the two locations of the system, but it was clearly smaller than the differences observed by Fizeau. It should be noted, incidentally, that by one of the experimental series, the heliostat mirror was replaced by a totally reflecting prism and that this seems to have been without influence on the results.

Everything taken together, the question is forced upon us, whether it might be possible to adapt the theory to observations, without ceasing to explain the other phenomena discussed in this work. I haven't succeeded in this, and I must therefore leave the whole question open, in the hope that others might overcome the difficulties that still exist.

That the improvement of the theory will not be so easy, and that the phenomena in the experiments of Fizeau in any case did not happen in the way, as they were interpreted by him in his introductory observations, this is what I finally want to show.

It will suffice to consider for this purpose, a single glass plate. If we decompose the velocity of translation in two component that are perpendicular to the plate, or parallel, then, if we neglect magnitudes of second order, the effects of those components remain side by side. The problem can thus be reduced to two simpler cases. It is now possible, without making special assumptions about the nature of light oscillations, that a translation perpendicular to the plate, can *not* have the expected influence of Fizeau; we will derive some general considerations. As for the other direction of translation, we can not speak so determined; it can only be shown, that the moving plate certainly not behaves like a stationary one of somewhat different refractive index.

§ 96. We consider two isotropic media (separated from each other by a plane) whose ponderable parts are either at rest, or move with a common velocity $\mathfrak{p}$ in a direction perpendicular to the marginal surface. If one part of this surface, whose dimensions are considerably larger than the wavelength, is hit by plane waves, which are laterally limited by a cylinder that shares the translation, then the reflection and refraction give rise to two similar light beams. Any theory of aberration has to assume now that, independent of translation, the describing lines of the cylindrical marginal-surfaces, the *relative* light rays, are subject to the ordinary laws of reflection and refraction.

Accordingly, we can once and for all imagine *four* cylinder: 1, 2, 3, 4, as those mentioned above, — we want to say "four *paths of light*" —, of which 1 and 2 are in the first, 3 and 4 in the second medium, and which belong together in the following way. From an incident motion in 1, a reflected one shall emerge in 2, and a transmitted one in 4, while also an incident beam in 3 gives rise to motions in 2 and 4. In reverse, incident oscillations in 2 or 4 will excite motions in paths 1 and 3.

For simplicity, we also assume [8] that the part of the marginal surface that was hit by light, has two symmetry axes that are mutually perpendicular, one of which lies in the plane of incidence of the rays. The figure consisting of four light paths, thus has two symmetry planes which go through one of these axes and the normal to the marginal surface. That one, which coincides with the plane of incidence, may be called the *first*, the other one be called the *second* symmetry plane.

§ 97. Of the deviations from equilibrium that constitutes light, it shall be assumed that they belong to *vector quantities*. If several such variables come into account, such as in the electro-magnetic theory of light: the dielectric polarization, the electric force, the magnetic force, or even the earlier vectors $\mathfrak{D}'$ and $\mathfrak{H}'$, then we have to imagine that for a given body (at a given beam direction, relative oscillation period and translation) these vectors were all determined by one of them. Thus it will be sufficient, to choose *one* of the vectors for consideration. This we call the *light vector* and introduce the following presuppositions, which partly includes a hypothesis about the nature of bodies and light, and partly a limitation in the choice of the light vector.

1°. If a state of motion exists in a system of bodies, in which the components of the light vector are certain functions of relative coordinates and time *t*, thus also the functions that arise when we replace *t* by *-t*, represents the values of the components that correspond to a possible motion. But, in the course of this reversal, we also have to reverse the motion and the velocity $\mathfrak{p}$.

2°. We also arrive at a possible motion, when we take the mirror image of an arbitrary, given motion in relation to a stationary plane, namely in such a way that both the translational velocity, as well as all light-vectors are replaced by the mirror images.

If we are dealing with the pure aether, then we satisfy these conditions if we choose the dielectric displacement as the light vector.

§ 98. In a *polarized* light beam, the light vector is parallel at all points to a certain line; it can be decomposed into three mutually perpendicular components, the first having the direction of the beam, while the second lies in the plane of incidence and the third is perpendicular to it. Now, since the properties of a polarized beam, except

the intensity and period of oscillation, only depends on *one* magnitude — such as the azimuths of the polarizer —, then the ratios between the mentioned components must have specific values, as soon as the ratio between the second and third is given; yet this *single* ratio must be allowed to have any arbitrary value. This can also be expressed as: If we decompose the light vector into two components, of which one has the direction of the beam, while the other is perpendicular to it, then the latter can be arbitrarily rotated around the beam, and in every direction, the ratio is determined between the two.

The state of motion is thus completely known, once the nature of the body, the translation, the relative period, the ray direction and finally the direction and magnitude of the "transverse" component of the light vector, are given. At the places where we will later speak of the light vector, we will only think of that transverse component.

Now, if this vector in the incident light is perpendicular to the plane of incidence, it must also have the same direction in the reflected and transmitted beams; in the same way, also the light vector in these beams must be parallel to the plane of incidence, as soon as the light vector of the incident light lies in that plane. To justify these theorems, we only have to consider the mirror image of the entire state of motion in relation to the first plane of symmetry. For example, the light vector of the incident light might have the first of those directions. In the transition to the mirror image, this vector gets the opposite direction, or, as it can also be said, the opposite phase; the light vector of the other two light beams now must be changed in the same way, hence the accuracy of the above claim follows immediately.

The problem is now reduced to the two main cases, *i.e.* that the light vectors are everywhere perpendicular to the plane of incidence, or are everywhere located in its interior. In the course of the further investigation, we always have to think of one of these cases; however, it applies to one case as well as to the others.

As regards each light path, we call a certain direction of the light vector positive, and namely, this direction shall be the same for all the light paths in the first main-case, while in the second main-case the positive directions chosen for 2 and 4 are mirror images of those adopted for 1 and 3 with respect to the second plane of symmetry.

Eventually, in order to represent the vibrations conveniently, we look at two points*P* and *Q*, which on both sides of the border area, lie in a fixed distance from it, at the intersection of two planes of symmetry.

Let *P* belong to the space, in which 1 and 2 are overlapping. Similarly, let *Q* simultaneously lie in 3 and 4. Only values of the light vectors in *P* and *Q* shall be given.

§ 99. If the light vector as regards incident motion has the value

$$q\ \cos\left(2\pi\tfrac{t}{T}+r\right)\ ,$$

it can be represented (for a reflected or transmitted beam that emerges from it) by

$$a\ q\ \cos\left(2\pi\tfrac{t}{T}+r-b\right)\ ,$$

where a and b are certain constants. In order to mutually distinguish the various cases, we want to append two indices on any of these magnitudes, the first of them is related to the path of the incident light, and the second is related to the beam that arose from it; additionally, also those a and b which remained without prime, are related to the case, when the translation is directed into the side of the incident light, while the primed letters apply to an equal and opposite displacement.

Let in light path 1 be an incident motion (while the system is progressing into the side of the first medium), at which the light vector has the value

$$\cos\ 2\pi\tfrac{t}{T}$$

From that, in 2 and 4 the light beams emerge which are represented by

$$a_{1.2}\ \cos\left(2\pi\tfrac{t}{T}-b_{1.2}\right)\ ,$$

and

$$a_{1.4}\ \cos\left(2\pi\tfrac{t}{T}-b_{1.4}\right)\ ,$$

Afterwards, we imagine this state of motion as reversed. First, we thus assume, that the translation is turned away from the first medium, and second, we replace *t* by *-t*. Then we find, that in 1 the light emerges

$$\cos\ 2\pi\tfrac{t}{T}$$

when in the paths 2 and 4 the incident motions

and

exist.

However, since the light vector, which is generated by the motion (120) in the first path, has the value

$$a_{1.2}a'_{2.1}\cos\left(2\pi\tfrac{t}{T}+b_{1.2}-b'_{2.1}\right)$$

and also the light vector emerging form (121), is to be replaced by

$$a_{1.4}a'_{4.1}\cos\left(2\pi\tfrac{t}{T}+b_{1.4}-b_{4.1}\right)\ ,$$

then it is given

From that if follows

and

§ 100. The following remark leads to a simple relation. If we start by a condition, at which the incident light follows path 1, and if we take the mirror image with respect to the second plane of symmetry (§ 96), then we arrive at a condition, at which the light is incident in 2. Consequently it has to be

and in the same way

For the difference $b_{1.2} - b'_{2.1}$ which comes into (123), we may put $b_{1.2} - b'_{1.2}$, which is evidently of order $\mathfrak{p}/V$, since the magnitudes $b_{1.2}$ and $b'_{1.2}$ are only different form one another by having different directions of translation.

By (123), also sin $(b_{1.4} - b_{4.1})$ must now be of order $\mathfrak{p}/V$. Since we additionally (without changing anything of the matter) can increase or decrease $b_{4.1}$ by a multiple of π , and also an uneven multiple of π as long as the sign of $a_{4.1}$ is reversed, then we may assume, that also the angle $b_{1.4} - b_{4.1}$ itself is of order $\mathfrak{p}/V$. The two cosines in (122) thus differ from unity only by magnitudes of second order, so that we may put

$$a_{1.2}\, a'_{2.1} + a_{1.4}\, a_{4.1} = 1 \, .$$

In the same way

$$a'_{1.2}\, a_{2.1} + a'_{1.4}\, a'_{4.1} = 1 \, ,$$

and under consideration of (124) and (125) we thus find

$$a_{1.4}\, a_{4.1} = a'_{1.4}\, a'_{4.1} \, .$$

Now suppose, similarly to the experiment of Fizeau, a plan-parallel glass plate (at whose two sides the aether is located) will be hit in oblique direction by a light beam, whose light vector has one of the directions previously distinguished, *i.e.* that it is polarized either in the plane of incidence, or perpendicular to it. The relation, be which the amplitude is diminished during the entrance, can thus be (depending in the direction of translation) represented by $a_{1.4}$ or $a'_{1.4}$, and also, as we can easily see, by the corresponding relation when leaving the plate by $a_{4.1}$ or $a'_{4.1}$. Altogether, the amplitude is thus altered in the ratio of 1 to $a_{1.4}\, a_{4.1}$ or $a'_{1.4}\, a'_{4.1}$. Now, since these products have the same value, the reversal of the translation changes nothing of the intensity of the leaving light, which consequently must be (except magnitudes of second order) the same, as if the plate would stand still: This is true for both main-positions of the polarization plane; consequently, when the incident rays are linearly polarized in an arbitrary way, the oscillation direction of the transmitted light is independent of the translation.

Here, it is to be noticed, that for the plane of incidence, as well as for the component polarized perpendicularly to the plane of incidence, we have to assume the dragging coefficient of Fresnel. Thus both are propagating with the same velocity, by which a phase difference between them and an elliptic polarization of the transmitted light is excluded.

§ 101. If the direction of translation is, as it was assumed in the last paragraph, not parallel to the marginal surface, but parallel to it, thus it must be distinguished, whether it lies in the plane of incidence, or perpendicular to it. We only want to consider the first case, and additionally restrict ourselves to the plane of incidence of polarized light.

At first it should be remembered, as to how we arrive to the value of the reflected amplitude for such light. If we choose the marginal surface with respect to *y z*-, and the plane of incidence with respect to the *x z*-plane, and we argue on the basis of the electromagnetic theory, then we have to put $\mathfrak{E}_x = \mathfrak{E}_z = 0$, and also $\mathfrak{H}_y = 0$, while the marginal conditions consist of the continuity $\mathfrak{E}_{y,}$ $\mathfrak{H}_x$ and $\mathfrak{H}_z$. Since in every of both media it is given by equation (IV_c) (§ 52)

$$\frac{\partial \mathfrak{H}_x}{\partial t} = \frac{\partial \mathfrak{E}_y}{\partial z} \text{ , und } \frac{\partial \mathfrak{H}_z}{\partial t} = -\frac{\partial \mathfrak{E}_y}{\partial x} \, ,$$

the the continuity of $\mathfrak{H}_x$ and $\mathfrak{H}_z$ has the same meaning as the continuity of $\partial\mathfrak{E}_y/\partial z$ and $\partial\mathfrak{E}_y/\partial x$. The first of those derivatives, however, will be steady, as soon as $\mathfrak{E}_y$ has this property itself, and at the end we are only dealing with $\mathfrak{E}_y$ and $\frac{\partial \mathfrak{E}_y}{\partial x}$.

Indeed — and this remark is true for every light theory — the known formula of Fresnel is given, when we assume, that this or that magnitude that come into consideration as regards to oscillations, and simultaneously its derivative with respect to the normal of the marginal surface, is steady.

As regards plane waves, the differentiation with respect to *x* amounts to the same, as if we would differentiate with respect to *t*, and then multiply by a factor *m* dependent on the direction and velocity of the waves. If we denote (for the incident, reflected and transmitted light) the values of the magnitude just mentioned in the immediate vicinity of the marginal surface by

$$\varphi_1(t), \varphi'_1(t) \text{ and } \varphi_2(t) \, ,$$

and the values of *m* by

$$m_1, m'_1 \text{ und } m_2$$

then we obtain as marginal conditions

$$\varphi_1(t) + \varphi'_1(t) = \varphi_2(t)$$

and

$$m_1 \frac{\partial \varphi_1(t)}{\partial t} + m'_1 \frac{\partial \varphi'_1(t)}{\partial t} = m_2 \frac{\partial \varphi_2(t)}{\partial t} \, .$$

The last formula leads — when we neglect additive constants — to

$$m_1\varphi_1(t) + m_1'\varphi_1'(t) = m_2\varphi_2(t) \,,$$

and it is further given by elimination of $\varphi_2(t)$

$$\varphi_1'(t) = \frac{m_1 - m_2}{m_2 - m_1'}\varphi_1(t) \,.$$

Now, that the amplitude of the reflected beam (at constant direction of the incident light) depends on the refractive index of the second body, stems from the fact, that, as it can easily be seen, m_2 changes with this exponent.

Now, in the next paragraph it should be demonstrated, that this m_2 (as long as the direction of the incident relative ray remains the same) is not affected by a translation in the direction of the z-axis. If it would be allowed to assume, that also with respect to a moving plate, the marginal conditions consist of the continuity of a certain magnitude φ and its derivatives, then, at least for light polarized in the plane of incidence, we would have demonstrated the impossibility of the phenomenon sought by Fizeau. However, in reality the assumption on the marginal conditions is not allowed without closer investigation; the things said show at least, however, that the moving plate in no ways acts as a stationary one of somewhat different refractive index.

§ 102. Let, with respect to the previously introduced axes,

$$\cos\alpha,\ 0,\ \sin\alpha$$

be the direction constants of the rays incident on the plate. Neglecting magnitudes of second order, we consequently obtain the direction of the wave normal by application of the fundamental law of aberration; namely we have to compose a velocity V in the direction of the rays with a translational velocity $\mathfrak{p}$. Now, if the latter is parallel to the z-axis, then the direction constants of the wave normal become,

$$\cos\alpha',\ 0,\ \sin\alpha'$$

where

$$\alpha' = \alpha + \frac{\mathfrak{p}_z}{V}\cos\alpha$$

The absolute velocity of the waves is V; however, the relative velocity V' will be found, when we diminish V by the component of $\mathfrak{p}$ with respect to the wave normal. If we understand by $x,\ y,\ z$ relative coordinates, then for the incident light, expressions of the form

$$A\ \cos\frac{2\pi}{T}\left(t - \frac{x\ \cos\alpha' + z\ \sin\alpha'}{V'} + B\right)$$

apply, or

On the other hand, for glass we have to assume Fresnel's dragging coefficient. Consequently, when we denote the propagation velocity in stationary glass by W, and the directions constants of the wave normal in the plate by

$$\cos\beta,\ 0,\ \sin\beta$$

we have to put for the relative velocity of the waves with respect to glass, by (82),

To light in the plate an expression applies now, which has the form:

and those will follow the incident oscillations in all points of the marginal surface, when the coefficient of z is the same as in formula (126).

Therefore we have

$$\sin\beta = \left(W - \mathfrak{p}_z \sin\beta\frac{W^2}{V^2}\right)\left(\frac{\sin\alpha}{V} + \frac{\mathfrak{p}_z}{V^2}\right) \,,$$

or we denote the refraction angle in the stationary plate by β_0 , so that

$$\sin\beta_0 = \frac{W}{V}\sin\alpha$$

$$\sin\beta = \sin\beta_0 + \frac{W\mathfrak{p}_z}{V^2}\cos^2\beta_0 \,.$$

From that it follows

However, for the factor which we above have called m_2 , the value is given by (128)

$$-\frac{\cos\beta}{W'} \,,$$

and this one, in consequence of (127) and (129), actually is independent of the translation.

[1] Mascart. Ann. de l'école normale, 2e sér., T. 1, pp. 210—214, 1872.

[2] Michelson. American Journal of Science, 3d Ser., Vol. 22, p. 120, 1881.

[3] Michelson and Morley. American Journal of Science, 3d Ser., Vol. 34, p. 333, 1887; Phil. Mag. 5th Ser., Vol 24, p. 449, 1887.

[4] Lorentz. Zittingsverslagen der Akad. v Wet. te Amsterdam, 1893—93, p. 74.

[5] As Fitzgerald was so friendly to tell me, that he dealt with this hypothesis already for a longer time in his lectures. In the literature, I only found it mentioned by Lodge, in the treatise „Aberration problems" (London Phil. Trans, Vol. 184, A, p. 727, 1893). I allow myself, to also add at his place, that this treatise, besides some theoretical considerations, also contains the description of very interesting experiments, in which two discs of metal (Diameter 1 Yard) perpendicularly fixed on that axis, were rotated with great velocity. By means of a certain interference method it was investigated, whether the aether that was present between the discs, was co-rotating; the result was negative, even though the number of rotations in a second was increased up to 20 or more. Lodge concludes, that the discs haven't communicated to the aether the 800th part of their velocity.

[6] see Lorentz, Arch. néerl., T. 21, pp. 168—176, 1887.

[7] Fizeau. Ann. de chim. et de phys., 3e sér., T. 58, p. 129. 1860; Pogg. Ann., Vol. 114, p. 554, 1861.

[8] This assumption can be dropped later, since the ratio of the intensities of the light beams is independent of the size and shape of the cross sections.

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2.1 Text

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